1.MEASURES OF CENTRAL TENDENCY

AVERAGE

- **Mathematical average**
  - Arithmetic mean
  - Geometric mean
  - Harmonic mean

- **Positional average**
  - Median
  - Mode
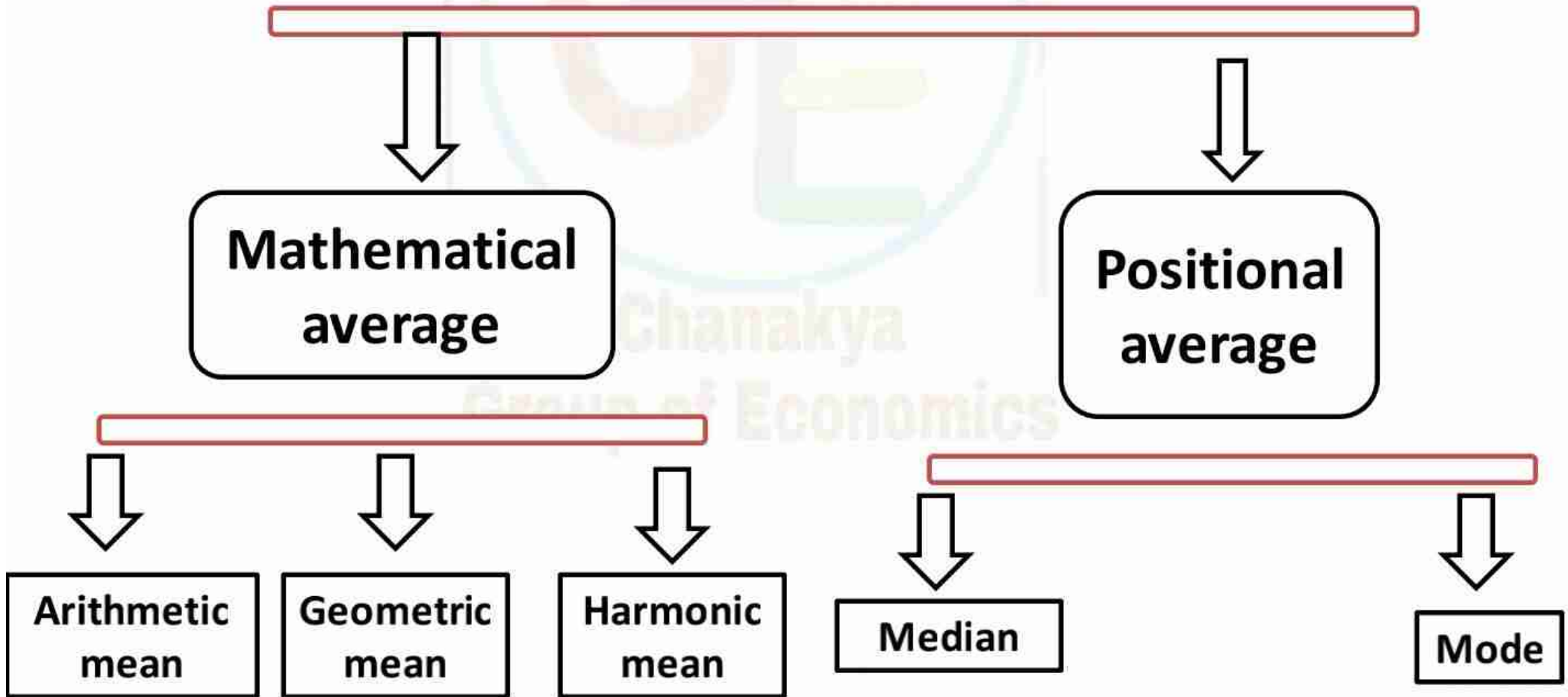
AVERAGE

An average is a single figure that represent whole group.

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                   ┌─────────────────────┐
                   │    Mathematical     │
                   │      average        │
                   └─────────────────────┘

   ┌──────────────────────────────────────────────────┐
   └──────────────────────────────────────────────────┘
        ↓                    ↓                    ↓
  ┌────────────┐      ┌────────────┐      ┌────────────┐
  │ Arithmetic │      │ Geometric  │      │  Harmonic  │
  │   mean     │      │    mean    │      │    mean    │
  └────────────┘      └────────────┘      └────────────┘
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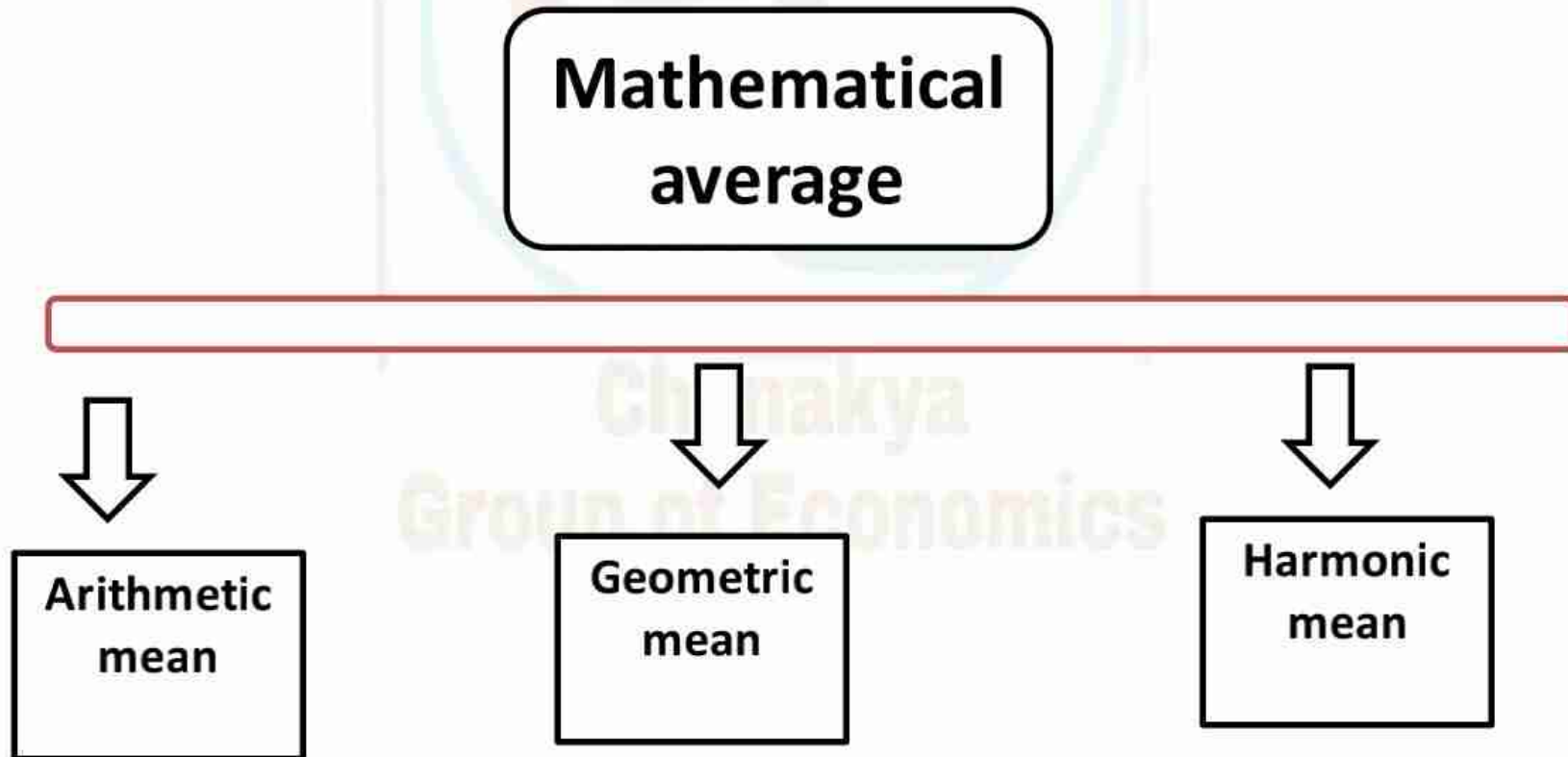
Mathematical average

A.Arithmetic mean-

Generally if we talk about average ,it signifies arithmetic mean.

It is also known as mean

Features-

It is based on all observations.

It is calculated value and not based on the position of the series.

Simple Arithmetic mean-

1.Direct method

$$\bar{X} = \frac{\sum X}{N}$$

$\bar{x}$ = Arithmetic mean.

$\sum X$-sum of the values of the item of a series.

N= number of observations.

2.Short cut method

$$\bar{X} = A + \frac{\sum d}{N}$$

$\bar{x}$ = Arithmetic mean.

A= Assumed mean.

$\sum d$= sum of deviation.

N= number of observations.

Properties of Arithmetic mean-

1. The sum of deviation of item from AM is always zero.

$$\sum(X-\overline{X})=0$$

2. The sum of squared deviation of AM is minimum.

B-Geometric Mean

GM define as n^{th} root of the product of all the n values of the variable.

$$GM = \sqrt[n]{X1.X2.X3\ldots\ldots Xn}$$

If there are two items we take square root .

Two items there and their values are 4 and 9.

GM will be- $GM = \sqrt[2]{4*9} = \sqrt{36} = 6$

if there are three items , we take **cube root.**

Three items and their values 2,4,8

$$GM = \sqrt[3]{2*4*8} = \sqrt{64} = 4$$

It is based on **all the items of the series.**

It gives less weight **to large items.**

It is best measure of **ratio change.**

The geometric mean is less than the arithmetic mean, G.M<A.M

The product of the items remains unchanged if each item is replaced by the geometric mean.

$\sqrt[3]{(15 \times 20 \times 0)} = \sqrt[3]{0}$ = infinity, and hence **inappropriate.**

Product of the series, 4,5, 20,25 = 10000

Geometric mean of the series = $\sqrt[4]{(4 \times 5 \times 20 \times 25)}$ = 10

The product of this series $10 \times 10 \times 10 \times 10 = 10000$. Proved.

C. Harmonic mean

HM is based on **reciprocal** of the items.

It is the reciprocal of the AM.

$$HM = \frac{N}{1/X_1 + 1/X_2 + 1/X_3 \quad .... + 1/X_n}$$

Harmonic mean can be computed from a series with any number of negative values.Thus, the harmonic mean of the values: -5 and -10 will be

$$\frac{2}{\frac{1}{-5} + \frac{1}{-10}} = \frac{2}{-3/10} = \frac{-20}{3} = -6.67$$

Harmonic mean cannot be computed from a series if any its values is zero.

This is because, the reciprocal of 0 does not exist i.e. = infinity.

For any series in which all the values are not equal nor any value is zero, the value of the harmonic mean is less than the geometric mean and arithmetic mean.

It is also called as – sub contrary mean.

2.Median

The median is that value of the variable which divides the group into **two equal parts.**

Median is determined by **first arranging the series** in an **ascending** or **descending** manner.

Median is denoted as **M**

$$M = \frac{N+1}{2}$$

1.If odd number series-

9,5,3,6,10,12,7.

Arrange items in either ascending or descending order.

Ser.no	Items
1	3
2	5
3	6
4	7
5	9
6	10
7	12

$$M= \frac{N+1}{2}$$

$$= \frac{7+1}{2} = 4$$

$$M= 7$$

1.If even number series-

12,16,14,18,24,20.

Arrange items in either ascending or descending order.

Ser.no	Items
6	24
5	20
4	18
3	16
2	13
1	12

$$M= \frac{N+1}{2}$$

$$= \frac{6+1}{2} = 3.5$$

$$= \frac{3^{rd}+4^{th}}{2} = \frac{18+16}{2} = \frac{34}{2}$$

$$M= 17$$

3.Mode

Mode is define as value which occur **most frequently.**

Mode is denoted as **Z.**

8,6,14,12,8,5,10,8,14,3,8,4.

$Z = 8$

Empirical Relationship b/w mean, median ,and mode.

In asymmetrical distribution the difference b/w $\bar{x}$ and z is 3 times the difference bw $\bar{x}$ and M.

$$\bar{x}-z = 3 (\bar{x} - M)$$

Mean

$$\bar{x}=1/2(3M-Z)$$

Median

$$M=1/3(2\bar{x}-z)$$

Mode

$$Z= 3M-2\bar{x}$$